

KP theory, total positivity and rational degenerations of M-curves

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Our purpose is the connection of two areas of mathematics:

- The theory of totally positive Grassmannians,
 - The rational degenerations of the M-curves,
- using the real finite-gap theory for solitons of the Kadomtsev-Petviashvili 2 (KP) equation.

In the first part of the talk, we explain how to associate to any KP soliton data in the real totally positive Grassmannian $Gr^+(k, n)$ the rational degeneration of an M-curve of genus $g = k(n - k)$ and the KP wavefunction. We remark that g is minimal for generic soliton data.

In the second part, we focus on the inverse problem of reconstructing the soliton data in $Gr^+(k, n)$ when G is a certain rational degeneration of a hyperelliptic M-curve of genus $(n - 1)$. We show that the KP reality condition in this case singles out a special family of KP multi-line solitons naturally connected to the finite non-periodic Toda hierarchy.

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