Symbolic methods for solving systems of linear ordinary differential equations

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The main purpose of this tutorial is to present and explain methods for studying systems of linear ordinary differential equations with emphasis on symbolic methods and their implementation in computer algebra systems.

Whether one is interested in global problems (finding closed form solutions, testing reducibility, computing properties of the differential Galois group) or in local problems (computing formal local invariants or local formal solutions) of linear differential scalar equations or systems, one has to develop and use appropriate tools for local analysis the purpose of which is to describe the behavior of the solutions near a given point without knowing these solutions in advance.

After introducing the basic tools of local analysis we present the state of the art of existing algorithms and programs for solving the main local problems such as determining the type of a given singularity, computing the rank of a singularity, computing the Newton polygon and Newton polynomials at a given singularity, finding the formal solution, etc. Next we explain how by piecing together the local information around the different singularities one can solve some global problems such as finding rational solutions, exponential solutions, factoring a given differential system, etc.