Orthogonal polynomials for the quartic potential: a case study for phases and Painlevé I and II transitions

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Resumen

In this talk I will review a detailed analysis of the orthogonal polynomials and corresponding recurrence coefficients for the varying weight $\exp\left[-N\left(\frac{1}{2}z^2 + \frac{1}{4}tz^4\right)\right]$ with integration supported on several arcs in the complex plane.

While some general results (Rakhmanov, Bertola, Kuijlaars-de Silva) are known for the equilibrium measure in the case of polynomial potentials of more general type, the detailed study of the "phase regions" in the complex parameter t can only be carried out on a case by case basis, and numerical computations are also (almost) necessary.

I will discuss these results (in collaboration with Alexander Tovbis, UCF, Florida) as well as some detailed information on the behaviour of the recurrence coefficients near the critical transition points, of which there are three of substantially different nature (two of them linked to the first Painlevé transcendent and the third to the second Painlevé transcendent).