Monodromy of semisimple Frobenius coalescent structures

Giordano Cotti

SISSA, Italy

Based on joint work with B. Dubrovin and D. Guzzetti. In occasion of the 1998 ICM in Berlin, B. Dubrovin conjectured an intriguing connection between the enumerative geometry of a Fano manifold X with algebro-geometric properties of exceptional collections in the derived category $D^b(X)$. Under the assumption of semisimplicity of the quantum cohomology of X, the conjecture prescribes an explicit form for local invariants of $QH^*(X)$, the so-called "monodromy data", in terms of Gram matrices and characteristic classes of objects of exceptional collections. Frobenius manifolds appearing in the study of the conjectural relations mentioned above typically show a coalescence phenomenon at points where the Frobenius algebra is semisimple, but the operator of multiplication by the Euler vector field has not simple spectrum. On the one hand, the definition of monodromy data is based on the analytic theory of isomonodromy deformations, which a priori cannot be applied at coalescence semisimple points of $QH^*(X)$. On the other hand, it turns out that the Frobenius structure may be known only at coalescence points, which are thus the only locus where the monodromy data can actually be computed. This is the case of the small quantum cohomology of complex Grassmannians, for which the occurrence and frequency of the coalescence phenomenon is surprisingly subordinate to the distribution of prime numbers. In this talk I will firstly show how under minimal conditions the classical theory of M. Jimbo, T. Miwa and K. Ueno (1981) can be extended to describe isomonodromy deformations at a coalescing irregular singularity; I will also show how to locally describe the Frobenius structure near coalescing semisimple points, and finally, what is the "mirror counterpart" of our description in terms of exceptional collections in the derived category.