Painlevé functions, Fredholm determinants and combinatorics

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I am going to explain explicit construction of solutions of monodromy preserving deformation equations in the Fuchsian setting. We will start by deriving Fredholm determinant representation for isomonodromic tau functions of Fuchsian systems with n regular singular points on the Riemann sphere and generic monodromy in $GL(N, C)$. The corresponding integral operator acts in the direct sum of $N(n-3)$ copies of $L^2(S^1)$. Its kernel will be expressed in terms of fundamental solutions of $n-2$ elementary 3-point Fuchsian systems whose monodromy is determined by monodromy of the relevant n-point system via a decomposition of the punctured sphere into pairs of pants.

In the rank $N = 2$ case, these building blocks have hypergeometric representations, the kernel becomes completely explicit and, being rewritten in Fourier basis, is given by an infinite Cauchy matrix. I am going to show that the principal minor expansion of the Fredholm determinant then yields a multivariate series representation for the tau function of the Garnier system obtained earlier via its identification with Fourier transform of Liouville conformal block (or a dual Nekrasov-Okounkov partition function). Further specialization to $n = 4$ will provide an explicit series representation of the general solution to Painlevé VI equation.