Mean values and heat type equations

Grzegorz Łysik

Jan Kochanowski University in Kielce, Poland

After introducing integral means over spheres and balls we derive the Pizzetti formulas for real analytic functions. Next we give applications of the Pizzetti formulas to the study of convergence and Borel summability of formal solutions to the classical heat equation and its some generalizations. We also give a characterization of real analytic functions in terms of integral means. The characterization justifies introduction of a definition of analytic functions on metric measure spaces.

In the second part we introduce integral mean value functions which are averages of integral means over spheres or balls and over their images under the action of a discrete group of complex rotations. In the case of real analytic functions we derive higher order Pizzetti's formulas. As applications we get:
• a maximum principle for polyharmonic functions;
• a characterization of functions of Laplacian growth;
• a characterization of convergent solutions to the initial value problem for the higher order heat type equation $\partial_t u = \Delta^p u$, $u(0,\cdot) = \varphi$ where $p \in \mathbb{N}$ and φ is real analytic;
• a Dirichlet type problem for polyharmonic functions.